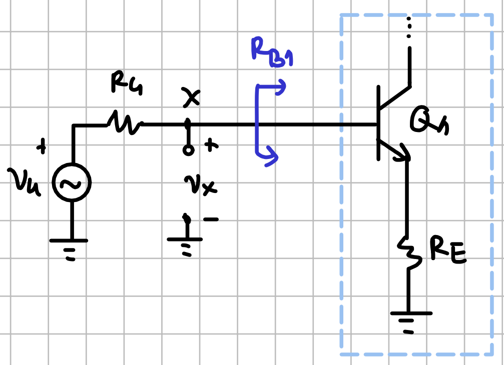
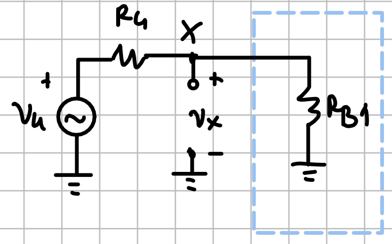
Пресликавање импеданси

Еквивалентне импедансе транзисторских кола:

Идеја 1: Читава кола која садрже отпорне мреже и транзисторе се могу заменити својом еквивалентном импедансом. Заменом кола еквивалентном импедансом на погодним местима се много лакше може одредити улазна или излазна импеданса појачавача или неки напони и струје у колу појачавача.



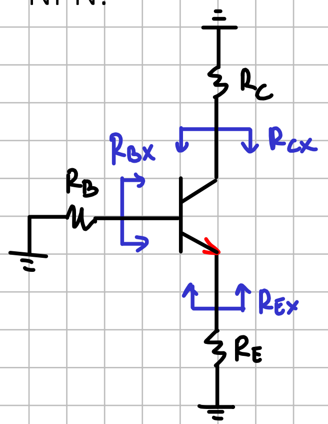
=>



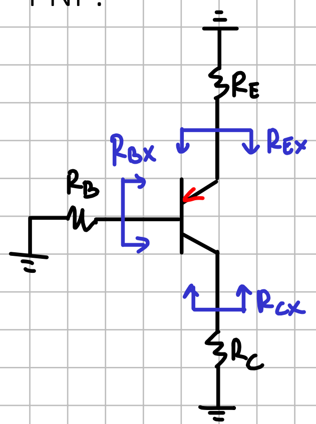
=> $$v_x = \frac{R_{B1}}{R_u + R_{B1}} \cdot v_u$$

Идеја 2: Може се приметити да се у колима појачавача често појављују једне те исте конфигурације транзистора (заједнички емитор, колектор итд.). Могу се извести изрази за екв. импедансе тих конфигурација који се касније могу директно примењивати у анализи појачавача. Тако много лакше можемо одредити појачање или улазне/излазне импедансе.

NPN:



PNP:



Јако битно !!! →

Ако је $r_o \to \infty$, важи:

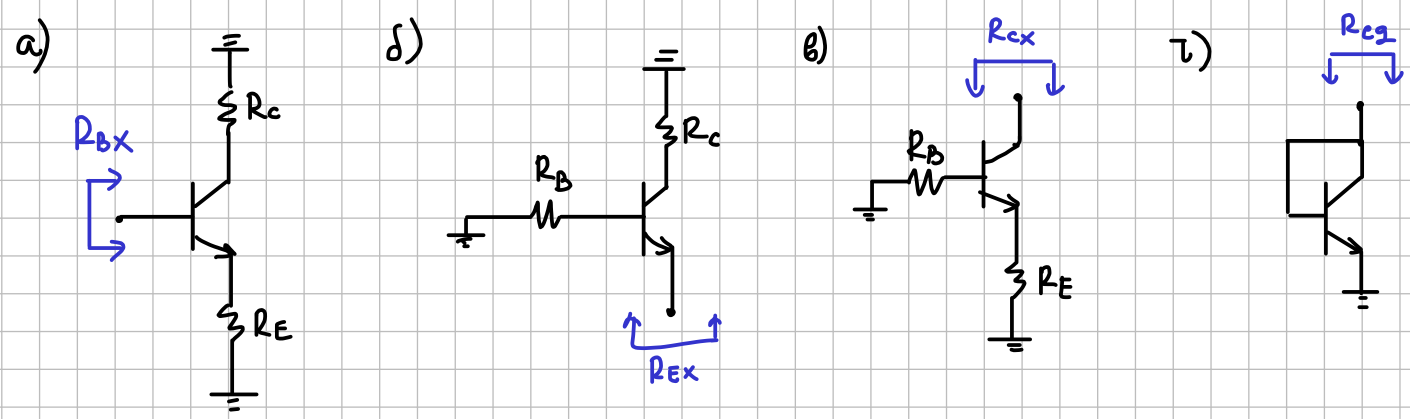
$$R_{BX} = r_\pi + (\beta+1)\cdot R_E$$

$$R_{CX} = \infty$$

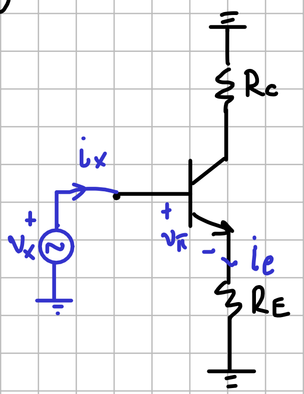
$$R_{EX} = \frac{r_\pi + R_B}{\beta + 1}$$

Исте једначине важе и за NPN и за PNP, пошто су и њихови модели за мале сигнале исти.

1. За дата кола одредити еквивалентне отпорности за мале сигнале (занемарити Ерлијев ефекат):



а)

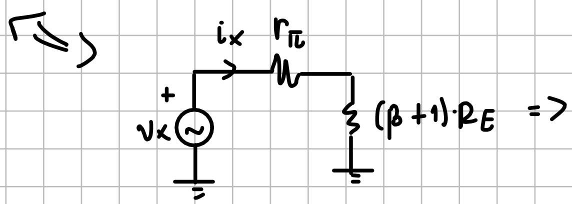


$$i_e = (\beta+1)\cdot i_x$$

$$v_\pi = r_\pi \cdot i_x$$

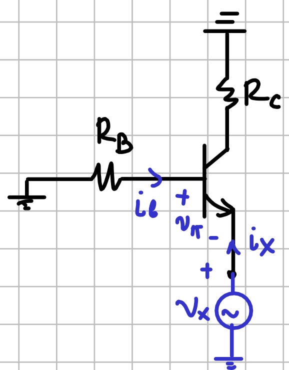
$$v_x = v_\pi + R_E \cdot i_e = r_\pi \cdot i_x + R_E \cdot (\beta+1)\cdot i_x = [r_\pi + (\beta+1)\cdot R_E]\cdot i_x$$

$$\boxed{R_{Bx} = \frac{v_x}{i_x} = r_\pi + (\beta+1)\cdot R_E}$$



$$\Rightarrow \quad v_x = r_\pi \cdot i_x + (\beta+1)\cdot R_E \cdot i_x$$

б)

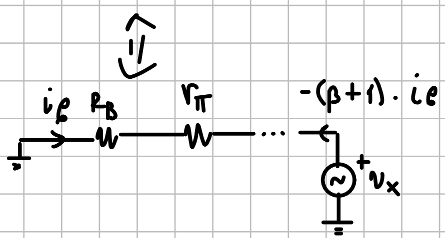


$$i_x = -i_e = -(\beta+1)\cdot i_b$$

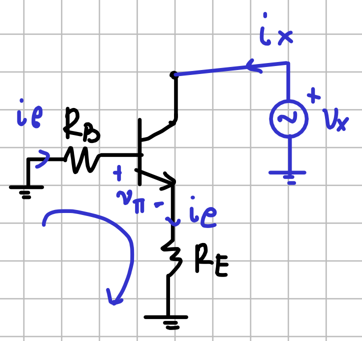
$$v_x = -v_\pi - R_B \cdot i_b = -r_\pi \cdot i_b - R_B \cdot i_b = -(R_B + r_\pi)\cdot i_b$$

$$R_{Ex} = \frac{v_x}{i_x} = \frac{v_x}{i_b}\cdot\frac{i_b}{i_x} = \frac{-(R_B + r_\pi)}{-(\beta+1)}$$

$$\boxed{R_{Ex} = \frac{R_B + r_\pi}{\beta+1}}$$



β)



$$R_B \cdot i_b + v_\pi + R_E \cdot i_e = 0$$

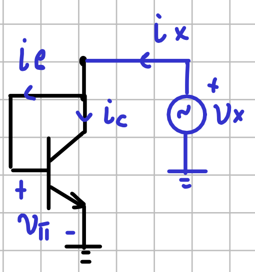
$$R_B \cdot i_b + r_\pi \cdot i_b + R_E \cdot (\beta+1) \cdot i_b = 0$$

$$(R_B + r_\pi + R_E \cdot (\beta+1)) \cdot i_b = 0 \Rightarrow i_b = 0$$

$$i_x = \beta \cdot i_b = 0$$

$$\boxed{R_{cx} = \frac{v_x}{i_x} \to \infty \quad (i_x = 0)}$$

γ)



$$i_x = i_b + i_c = i_e = (\beta+1) \cdot i_b$$

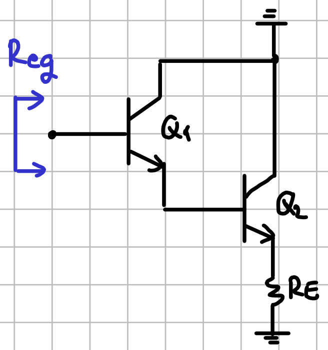
$$v_\pi = v_x = r_\pi \cdot i_b$$

$$R_{e2} = \frac{v_x}{i_x} = \frac{r_\pi \cdot \cancel{i_b}}{(\beta+1) \cdot \cancel{i_b}} = \frac{r_\pi}{\beta+1}$$

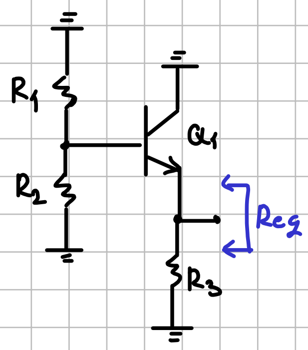
$$\boxed{R_{e2} = \frac{r_\pi}{\beta+1} \underset{(\beta \gg 1)}{\approx} \frac{r_\pi}{\beta} = \frac{\cancel{r_\pi}}{g_m \cdot \cancel{r_\pi}} = \frac{1}{g_m}}$$

2. Користећи претходно добијене резултате, за дата кола одредити еквивалентне отпорности за мале сигнале. Занемарити Ерлијев ефекат и узети $\beta_1 = \beta_2 = \beta$

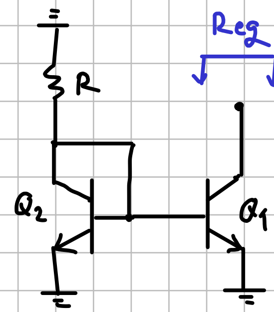
а)



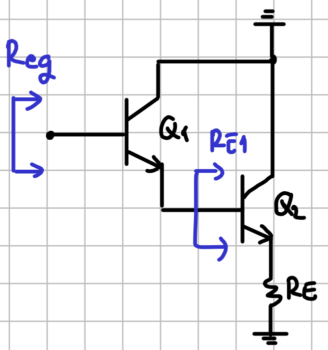
б)



в)



а)

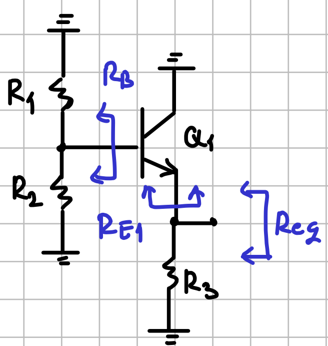


$$R_{eq} = r_{\pi 1} + (\beta+1)\cdot R_{E1}$$

$$R_{E1} = r_{\pi 2} + (\beta+1)\cdot R_E$$

$$\boxed{R_{eq} = r_{\pi 1} + (\beta+1)\left[r_{\pi 2} + (\beta+1) R_E\right]}$$

б)



$$R_{eq} = R_3 \parallel R_{E1}$$

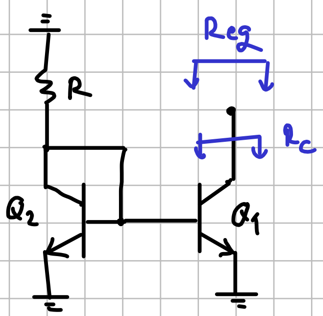
$$R_B = R_1 \parallel R_2$$

$$R_{E1} = \frac{R_B + r_\pi}{\beta+1}$$

$$\Rightarrow \boxed{R_{eq} = R_3 \parallel \left[\frac{R_1 \parallel R_2 + r_\pi}{\beta+1}\right]}$$

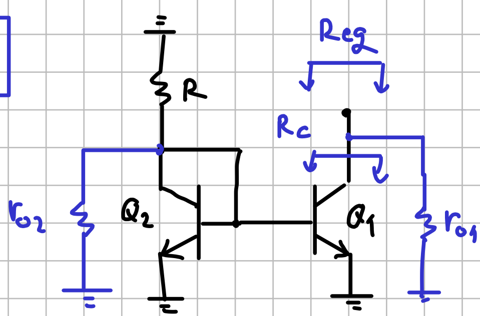
в)

$r_o \to \infty$



$$\boxed{R_{eq} = R_c = \infty}$$

$r_o < \infty$ $\quad r_{o1} = r_{o2} = r_o$



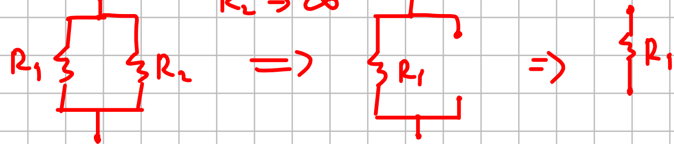
$$R_{eq} = R_c \parallel r_{o1}$$

$$R_c \to \infty$$

$$\Rightarrow \boxed{R_{eq} = r_{o1}}$$

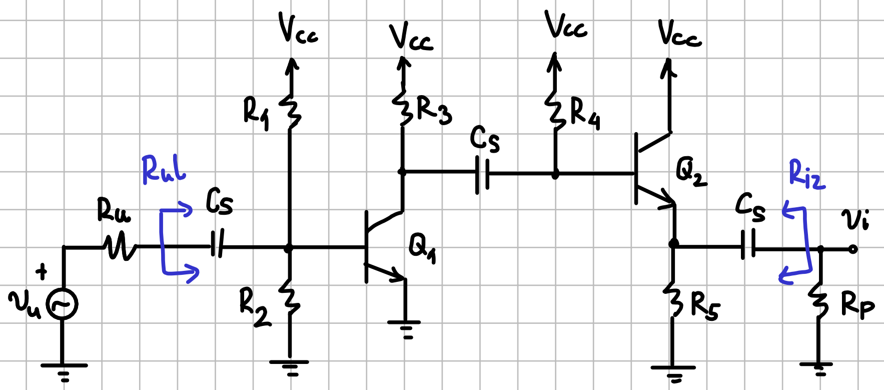
$R_1 < \infty$

$R_2 \to \infty$



$$R_{eq} = R_1 \parallel R_2 = R_1$$

3.



$V_{A1}, V_{A2} \to \infty, \quad C_S \to \infty$

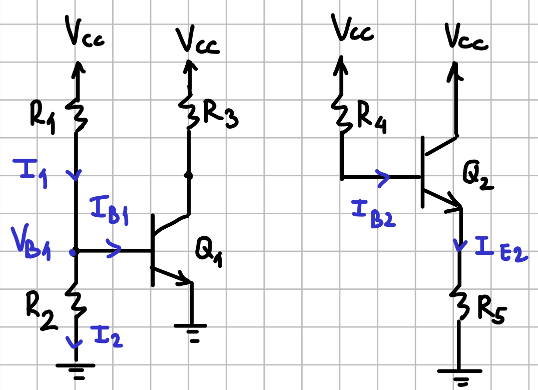
$\beta_1 = \beta_2 = \beta$

a) $g_{m1}, r_{\pi1}, g_{m2}, r_{\pi2} = ?$

б) $A_n = ?$

в) $R_{ul}, R_{iz} = ?$

a) DC:



$V_{B1} = V_{BE}$

$I_{B1} = I_1 - I_2$

$= \dfrac{V_{CC} - V_{B1}}{R_1} - \dfrac{V_{B1}}{R_2} = \dfrac{V_{CC} - V_{BE}}{R_1} - \dfrac{V_{BE}}{R_2} = \dots$

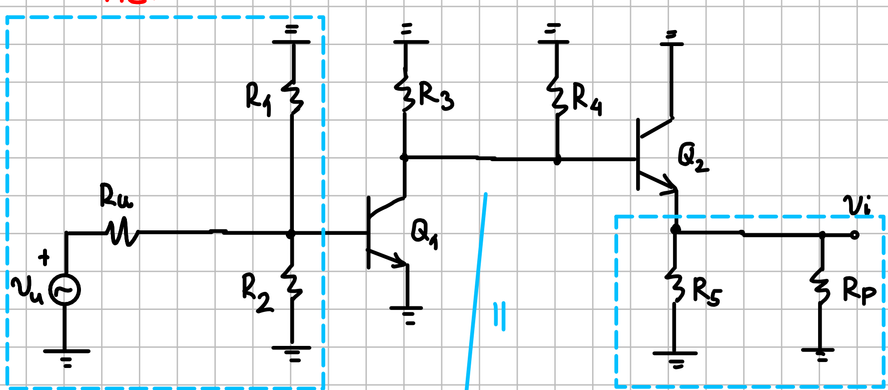
$I_{C1} = \beta \cdot I_{B1}$

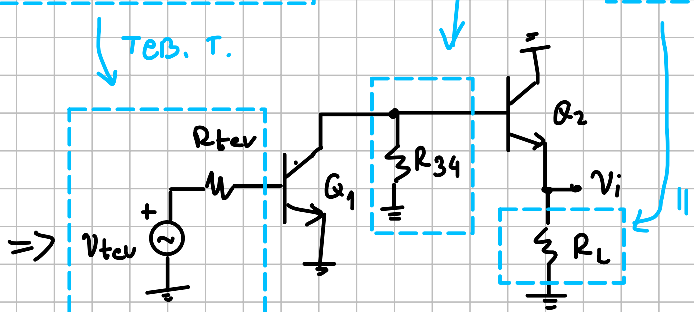
$r_{\pi1} = \dfrac{V_T}{I_{B1}} = \dots \qquad g_{m1} = \dfrac{I_{C1}}{V_T} = \dfrac{\beta}{r_{\pi1}} = \dots$

$$\left.\begin{aligned} R_4 \cdot I_{B2} + V_{BE} + R_5 \cdot I_{E2} &= V_{CC} \\ I_{E2} &= (\beta+1) \cdot I_{B2} \end{aligned}\right\} \Rightarrow I_{B2} = \frac{V_{CC} - V_{BE}}{R_4 + (\beta+1) R_5} = \dots \qquad I_{C2} = \beta \cdot I_{B2} = \dots$$

$\Rightarrow r_{\pi2} = \dfrac{V_T}{I_{B2}} = \dots \qquad g_{m2} = \dfrac{I_{C2}}{V_T} = \dfrac{\beta}{r_{\pi2}} = \dots$

б) AC:



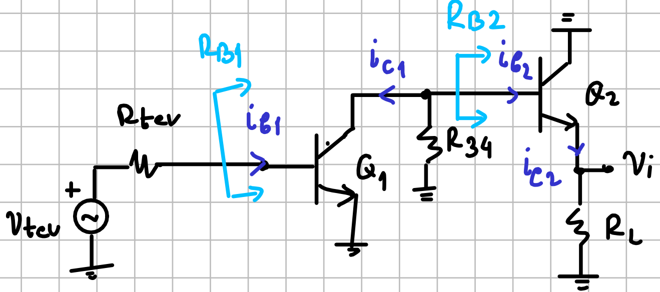


$R_{tev} = R_u \parallel R_1 \parallel R_2$

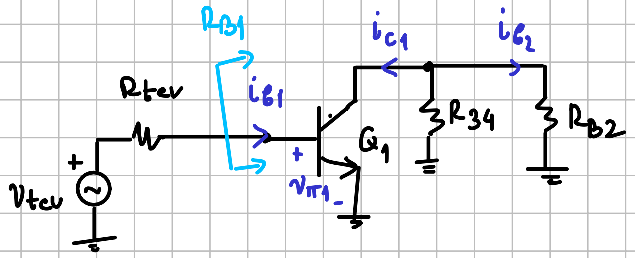
$v_{tev} = \dfrac{R_1 \parallel R_2}{R_1 \parallel R_2 + R_u} \cdot v_u$

$R_{34} = R_3 \parallel R_4 = \dfrac{R_3 \cdot R_4}{R_3 + R_4}$

$R_L = R_5 \parallel R_p = \dfrac{R_5 \cdot R_p}{R_5 + R_p}$



⇕



$$A_n = \frac{v_i}{v_u} = \frac{v_i}{i_{e2}} \cdot \frac{i_{e2}}{i_{b2}} \cdot \frac{i_{b2}}{i_{c1}} \cdot \frac{i_{c1}}{i_{b1}} \cdot \frac{i_{b1}}{v_{tev}} \cdot \frac{v_{tev}}{v_u}$$

$$R_{B1} = r_{\pi 1} \left(+ (\beta+1)\cdot 0\right) = \dots$$

$$R_{B2} = r_{\pi 2} + (\beta+1)\cdot R_L = \dots$$

$$\frac{v_i}{i_{e2}} = R_L \qquad \frac{i_{e2}}{i_{b2}} = \beta + 1$$

$$\Rightarrow \quad \frac{i_{b2}}{i_{c1}} = \frac{-R_{34}}{R_{34} + R_{B2}}$$

$$\frac{i_{c1}}{i_{b1}} = \beta$$

$$v_{tev} = R_{tev}\cdot i_{b1} + v_{\pi 1}$$

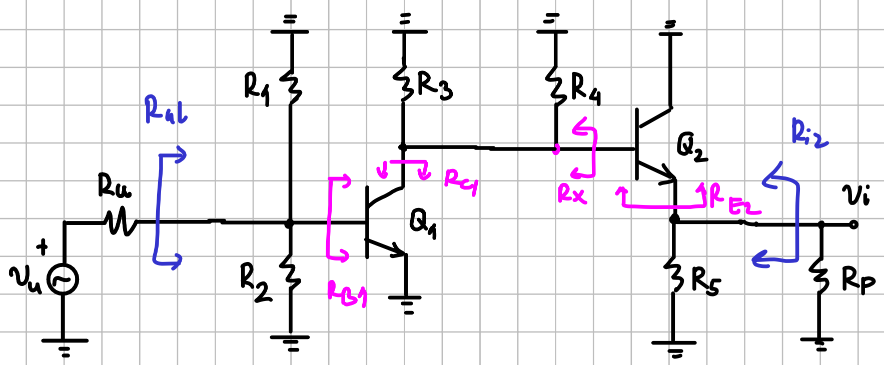
$$= R_{tev}\cdot i_{b1} + r_{\pi 1}\cdot i_{b1}$$

$$= (R_{tev} + r_{\pi 1})\cdot i_{b1}$$

$$\Rightarrow \quad \frac{i_{b1}}{v_{tev}} = \frac{1}{R_{tev} + r_{\pi 1}} \qquad \frac{v_{tev}}{v_u} = \frac{R_1 \parallel R_2}{R_1 \parallel R_2 + R_u}$$

$$A_n = R_L \cdot (\beta+1) \cdot \frac{-R_{34}}{R_{34} + R_{B2}} \cdot \beta \cdot \frac{1}{R_{tev} + r_{\pi 1}} \cdot \frac{R_1 \parallel R_2}{R_1 \parallel R_2 + R_u} = \dots$$

) AC:



$$R_{ul} = R_1 \parallel R_2 \parallel R_{B1}$$

$$R_{i2} = R_5 \parallel R_{E2}$$

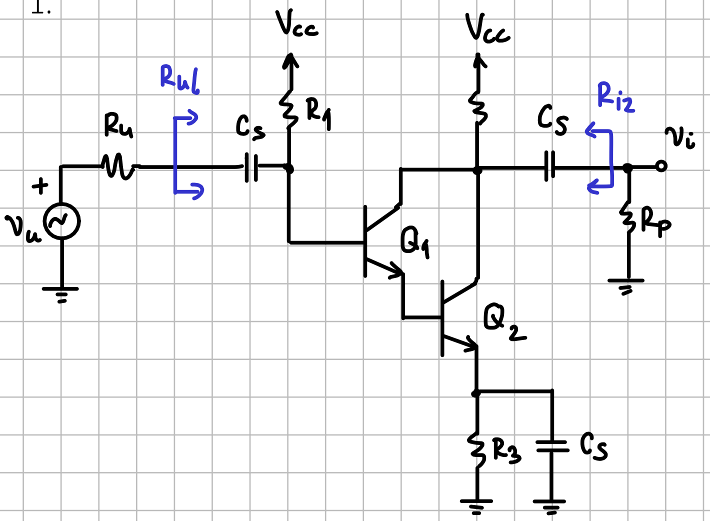
$$R_{E2} = \frac{r_{\pi 2} + R_x}{\beta + 1} \qquad R_x = R_3 \parallel R_4 \parallel R_{c1}$$

$$R_{c1} = \infty$$

$$\Rightarrow R_x = R_3 \parallel R_4$$

За вежбу:

1.



$V_{A1}, V_{A2} \to \infty, \quad C_S \to \infty$

а) $g_{m1}, r_{\pi 1}, g_{m2}, r_{\pi 2} = ?$

б) $A_n = ?$

в) $R_{ul}, R_{i2} = ?$

2.