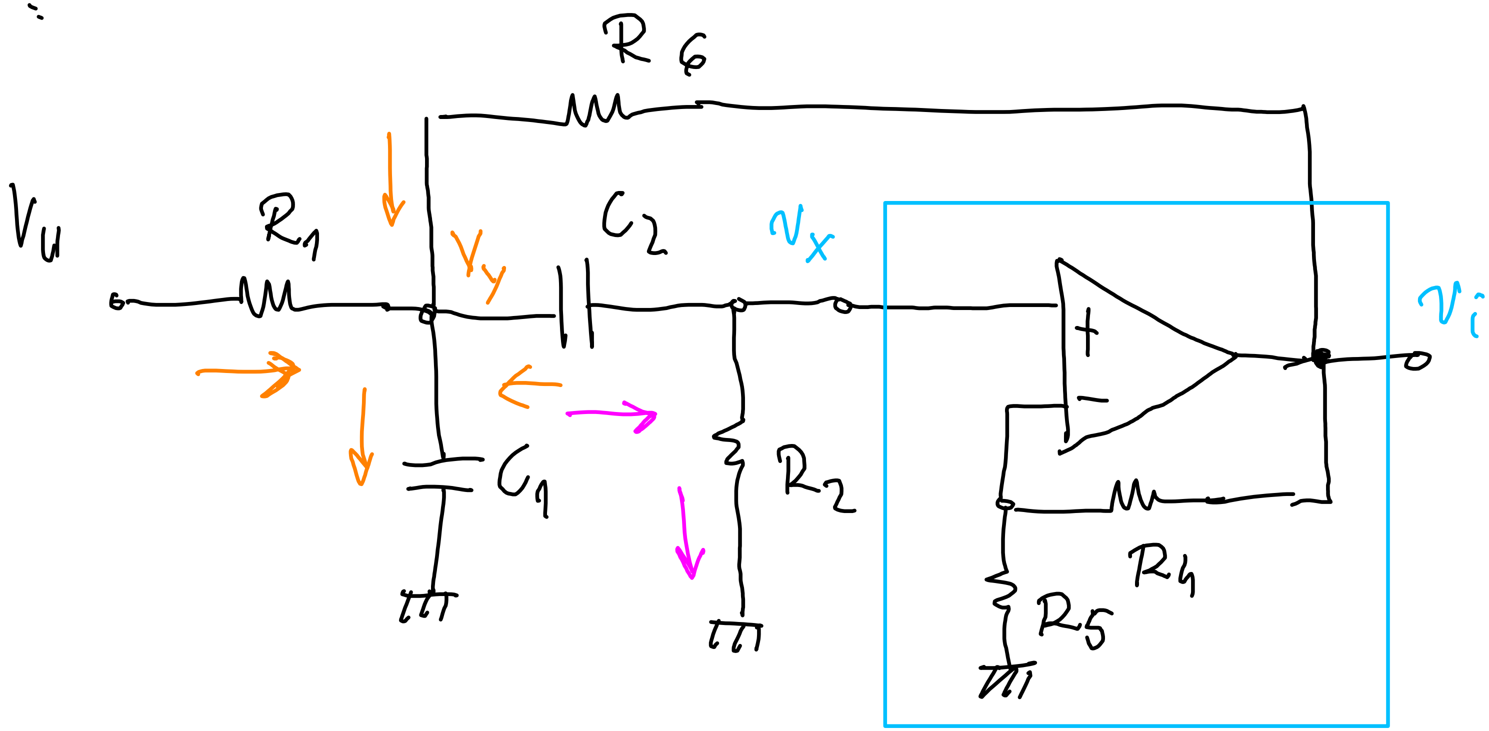
P 1



$$v_x = \frac{v_i}{K}$$

$$K = 1 + \frac{R_4}{R_5}$$

Y $\quad \dfrac{V_u - V_y}{R_1} + (V_x - V_y)\, sC_2 + \dfrac{V_i - V_y}{R_6} = V_y \cdot sC_1$

X $\quad (V_y - V_x) \cdot sC_2 = \dfrac{V_x}{R_2}$

LJ $\quad V_x = V_i / K$

$$\begin{array}{c} Y \\[2em] X \\[2em] H \end{array} \begin{bmatrix} -\left[\frac{1}{R_1}+\frac{1}{R_6}+s(C_2+C_1)\right] & sC_2 & \frac{1}{R_6} \\[1em] sC_2 & -\left(sC_2+\frac{1}{R_2}\right) & 0 \\[1em] 0 & 1 & -\frac{1}{K} \end{bmatrix} \begin{bmatrix} V_y \\[1em] V_x \\[1em] V_i \end{bmatrix} = \begin{bmatrix} -\frac{V_u}{R_1} \\[1em] 0 \\[1em] 0 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} \overset{+}{-\left[\frac{1}{R_1}+\frac{1}{R_6}+s(C_2+C_1)\right]} & \overset{-}{sC_2} & \overset{+}{\frac{1}{R_6}} \\ \overset{-}{sC_2} & \overset{+}{-\left(sC_2+\frac{1}{R_2}\right)} & \overset{-}{0} \\ \overset{+}{0} & \overset{-}{1} & \overset{+}{-\frac{1}{K}} \end{vmatrix} =$$

$$\Delta = \frac{1}{R_6}\cdot sC_2 - \frac{1}{K}\left[\left(\frac{1}{R_1}+\frac{1}{R_6}+s\left(C_2+C_1\right)\right)\cdot\left(sC_2+\frac{1}{R_2}\right) - s^2C_2^2\right]$$

$$\Delta = -\frac{s^2}{K}\left[C_1\cdot C_2\right] + s\left[\frac{1}{R_6}C_2 - \frac{C_2+C_1}{KR_2} - \frac{C_2}{K}\left(\frac{1}{R_1}+\frac{1}{R_6}\right)\right] - \frac{1}{K}\frac{1}{R_2}\left(\frac{1}{R_1}+\frac{1}{R_6}\right)$$

$$\Delta_{V_i} = \begin{vmatrix} -\left[\frac{1}{R_1}+\frac{1}{R_6}+s(C_2+C_1)\right] & sC_2 & -\frac{V_M}{R_1} \\ sC_2 & -\left(sC_2+\frac{1}{R_2}\right) & 0 \\ 0 & 1 & 0 \end{vmatrix} = -\frac{V_M}{R_1}\cdot sC_2$$

$$H(s) = \frac{V_i}{V_M} = \frac{\Delta V_i}{\Delta}\cdot\frac{1}{V_M} =$$

$$k R_1 \cdot R_2$$

$$\frac{-\frac{sC_2}{R_1}}{-\frac{s^2}{k}\left[C_1\cdot C_2\right] + s\left[\frac{1}{R_6}C_2 - \frac{C_2+C_1}{kR_2} - \frac{C_2}{k}\left(\frac{1}{R_1}+\frac{1}{R_6}\right)\right] - \frac{1}{k}\frac{1}{R_2}\left(\frac{1}{R_1}+\frac{1}{R_6}\right)}$$

$$\times k \cdot R_1 \cdot R_2$$

$$\frac{-sC_2R_2\cdot k}{-s^2C_1C_2R_1R_2+s\left[k\frac{R_1}{R_6}R_2C_2-R_1(C_2+C_1)-R_2C_2\left(1+\frac{R_1}{R_6}\right)\right]-\left(1+\frac{R_1}{R_6}\right)}$$

$$\frac{k\ sC_2R_2}{s^2C_1R_1C_2R_2+s\left[R_1(C_2+C_1)+R_2C_2\left(1+\frac{R_1}{R_6}\right)-R_2C_2k\cdot\frac{R_1}{R_6}\right]+\frac{R_1}{R_6}+1}$$

$$\frac{R_6}{R_6+R_1}\cdot\frac{k\ sC_2R_2}{1+s\left[R_2C_2+\frac{R_6}{R_6+R_1}\cdot\left(R_1(C_1+C_2)-k\cdot\frac{R_1}{R_6}\cdot R_2C_2\right)\right]+s^2C_1R_1C_2R_2\cdot\frac{R_6}{R_6+R_1}}$$

$$H(s) = H_0 \cdot \frac{s/\omega_1}{1 + a_1 \cdot s + a_2 \cdot s^2}$$

$$H_0 = k \cdot \frac{R_6}{R_6 + R_1} = \frac{1 + R_4/R_5}{1 + R_1/R_6} \quad ; \quad \omega_1 = \frac{1}{C_2 R_2} \left[\frac{rad}{s}\right]$$

$$a_1 = \left[R_2 C_2 + \frac{R_6}{R_6 + R_1} \cdot \left(R_1 (C_1 + C_2) - k \cdot \frac{R_1}{R_6} \cdot R_2 C_2 \right) \right] \quad [s]$$

$$a_2 = C_1 R_1 C_2 R_2 \cdot \frac{R_6}{R_6 + R_1} \quad [s^2]$$

$$R_1 = R_2 = R_6 = R \Rightarrow H_0 = \frac{k}{2} ; \omega_1 = \frac{1}{RC} ; a_1 = RC\,\frac{4-k}{2} ; a_2 = \frac{(RC)^2}{2}$$

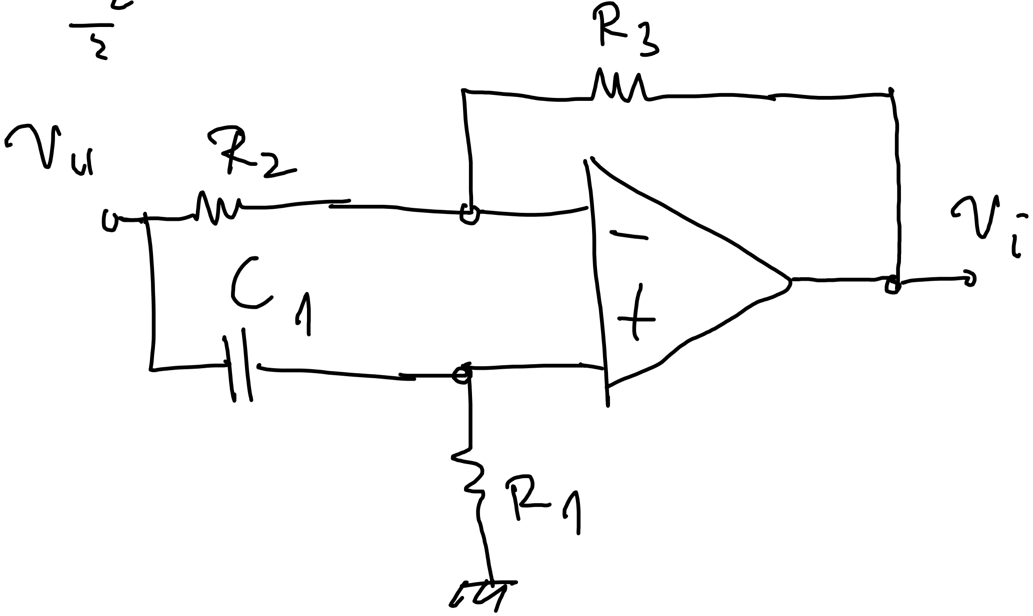
$$\boxed{a_2 = \frac{1}{\omega_0^2} \Rightarrow \omega_0 = \frac{\sqrt{2}}{RC}} \; ; \; \boxed{a_1 = \frac{1}{Q\omega_0} \Rightarrow Q = \frac{1}{a_1 \omega_0} = \frac{2}{(4-k)R\cancel{C}} \cdot \frac{\cancel{R}C}{\sqrt{2}} = \frac{\sqrt{2}}{4-k}}$$

$$|H| = \frac{H_0\, \omega/\omega_1}{\sqrt{\left(1 - (\omega/\omega_0)^2\right)^2 + \left(\frac{\omega}{Q\omega_0}\right)^2}}$$

$$\boxed{|H|\Big|_{\omega=\omega_0} = H_0 Q \frac{\omega_0}{\omega_1} = \frac{k}{2} \cdot \frac{\sqrt{2}}{4-k} \cdot \sqrt{2} = \frac{k}{4-k}}$$

P_2

2



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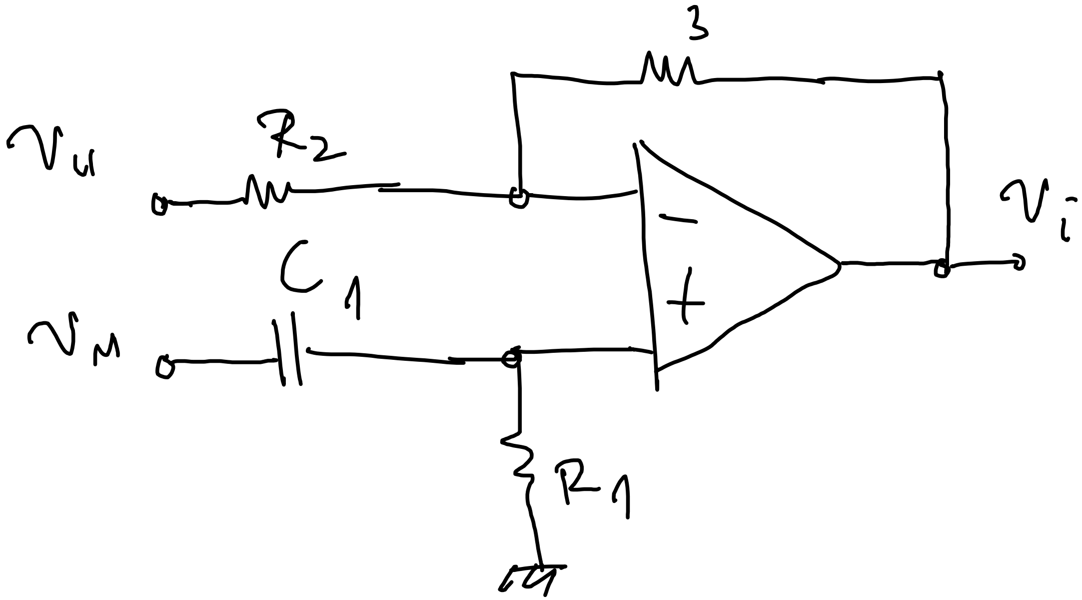
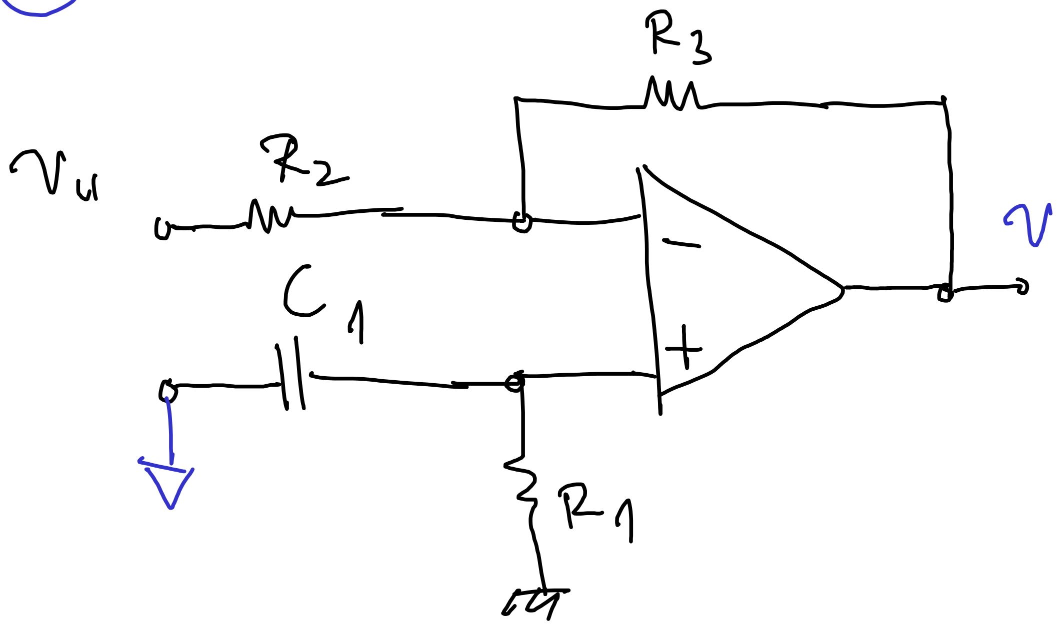
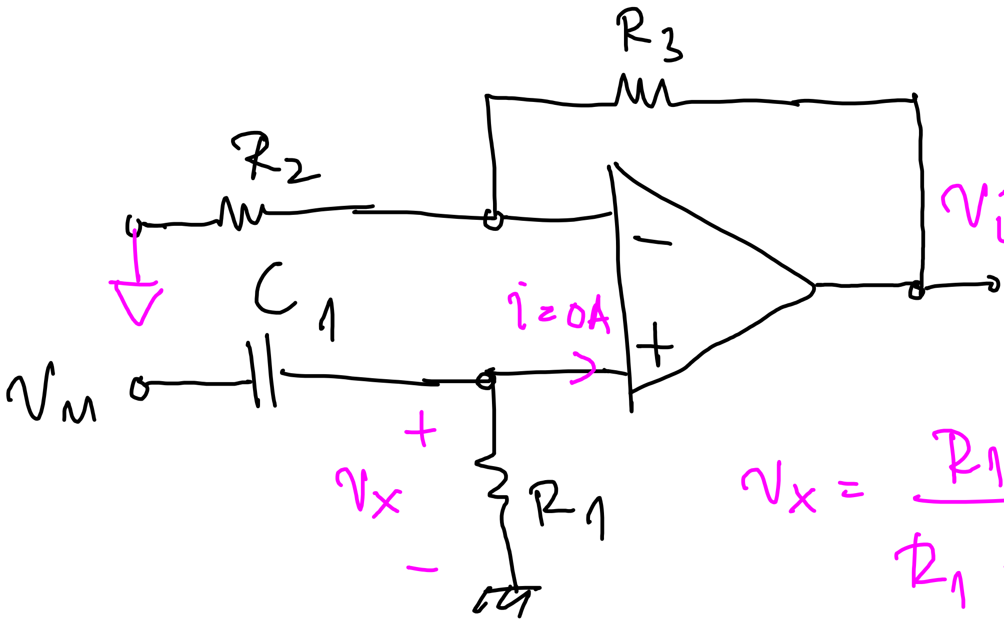


A



$$V_{i_A}^{-} = -\frac{R_3}{R_2} \cdot V_u$$



$$V_{i,B} = \left(1+\frac{R_3}{R_2}\right)V_x = \left(1+\frac{R_3}{R_2}\right)\cdot\frac{sC_1R_1}{1+sC_1R_1}V_u$$

$$V_x = \frac{R_1}{R_1+\frac{1}{sC_1}}\cdot V_u = \frac{sC_1R_1}{1+sC_1R_1}\cdot V_u$$

$$V_i = V_{i,A}+V_{i,B} = -\frac{R_3}{R_2}\cdot V_u + \left(1+\frac{R_3}{R_2}\right)\frac{sC_1R_1}{1+sC_2R_2}\cdot V_u$$

$$H(s)=\frac{V_i}{V_u} = \frac{-R_3/R_2 - \frac{R_3}{\cancel{R_2}}sC_2\cancel{R_2} + \left(1+\frac{R_3}{R_2}\right)\cdot sC_1R_1}{1+sC_2R_2}$$

$$H(s) = -\frac{R_3}{R_2}\,\frac{1+s\left[C_2R_3-\left(1+\frac{R_3}{R_2}\right)C_1R_1\right]}{1+sC_2R_2}$$

$$R_1 = R_2 = R_3 = R \; ; \quad C_2 = C$$

$$H(s) = -\frac{1 - sRC}{1 + sRC} = -\frac{1 - s/\omega_1}{1 + s/\omega_1}$$

$$|H|_{dB} = 0\,dB = 20\log_{10} \frac{\sqrt{1+(\omega/\omega_1)^2}}{\sqrt{1+(\omega/\omega_1)^2}}$$

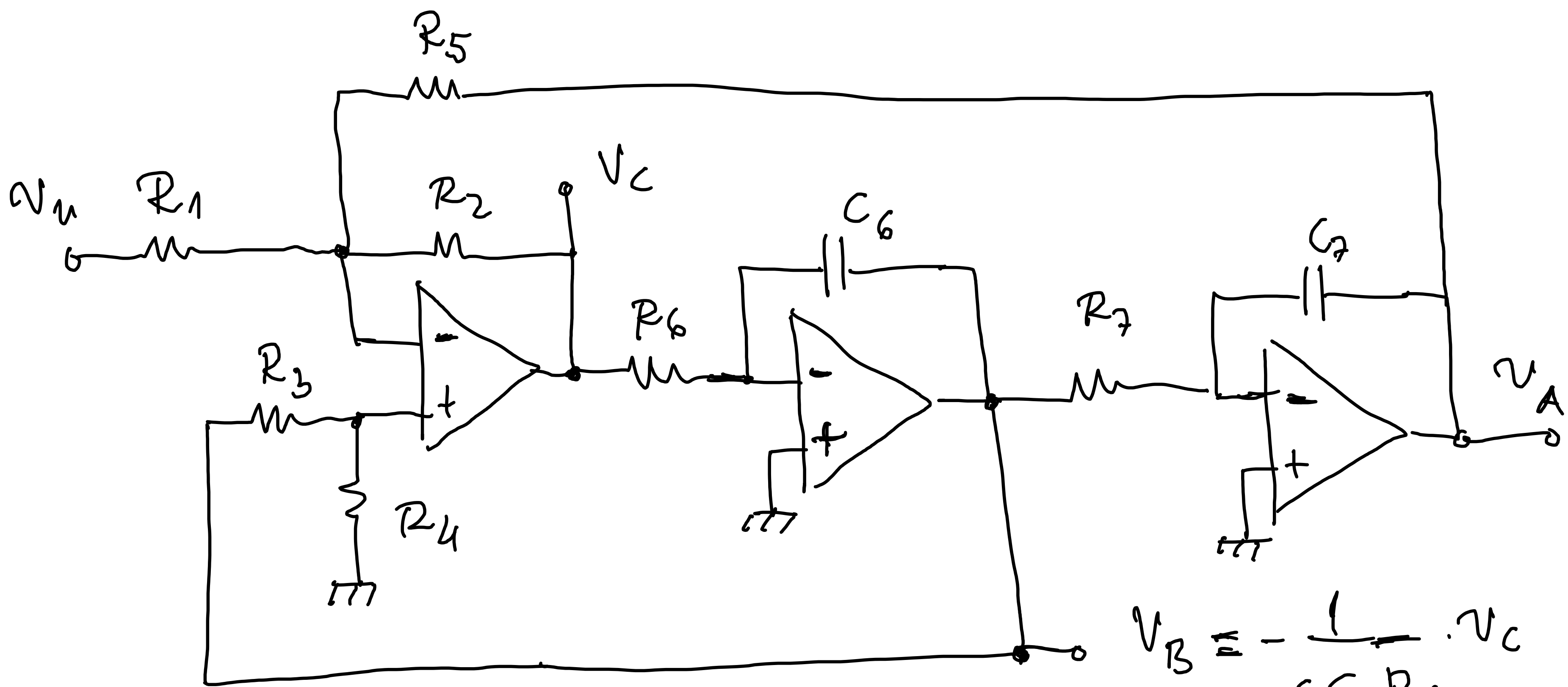
$$\angle H = -\pi + \arctan\left(-\frac{\omega}{\omega_1}\right) - \operatorname{arctg}\left(\frac{\omega}{\omega_1}\right)$$

$$\angle H = -\pi - 2\arctan\left(\frac{\omega}{\omega_1}\right)$$

$$\angle H[^\circ] = 180^\circ - 2\cdot\arctan\left(\frac{\omega}{\omega_1}\right) \Rightarrow \angle H\Big|_{\omega > 10\omega_1} \cong 0^\circ$$

$$\angle H[^\circ] = -180^\circ - 2\cdot\arctan\left(\frac{\omega}{\omega_1}\right) \Rightarrow \angle H\Big|_{\omega > 10\omega_1} \cong -270^\circ$$

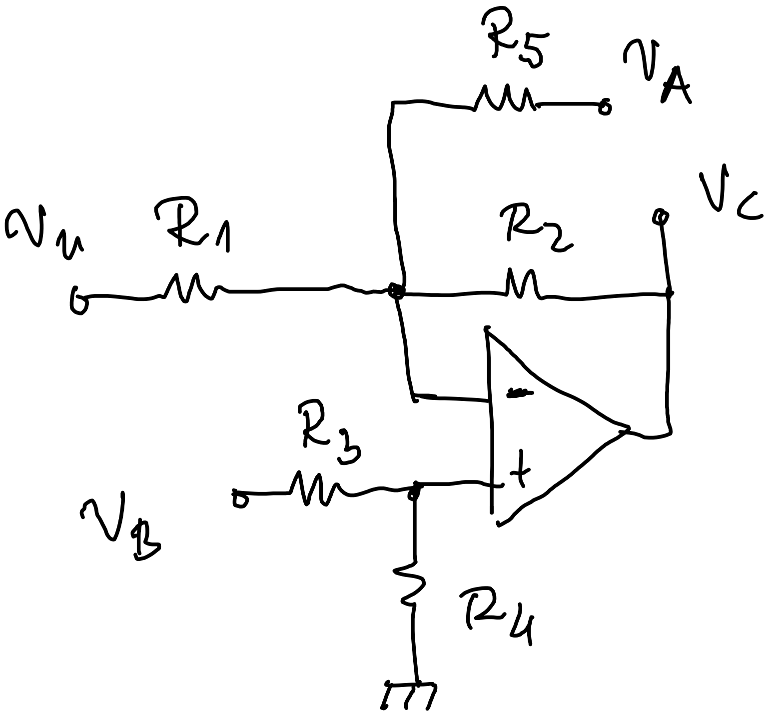
P_3



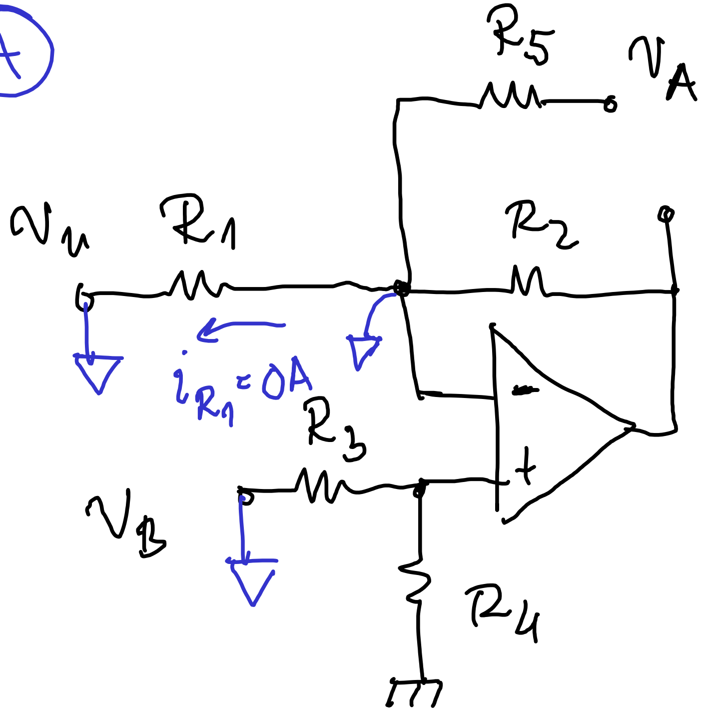
$$v_A = -\frac{1}{sC_7R_7}\cdot v_B$$

$$v_B = -\frac{1}{sC_6R_6}\cdot v_C$$

$$v_A = \frac{1}{s^2 R_6 C_6 R_7 \cdot C_7}\cdot v_C$$

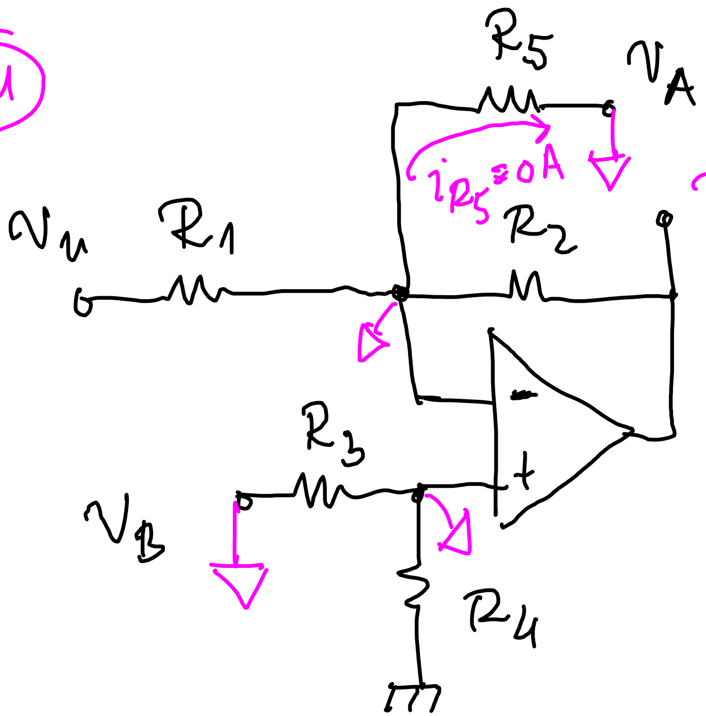


A



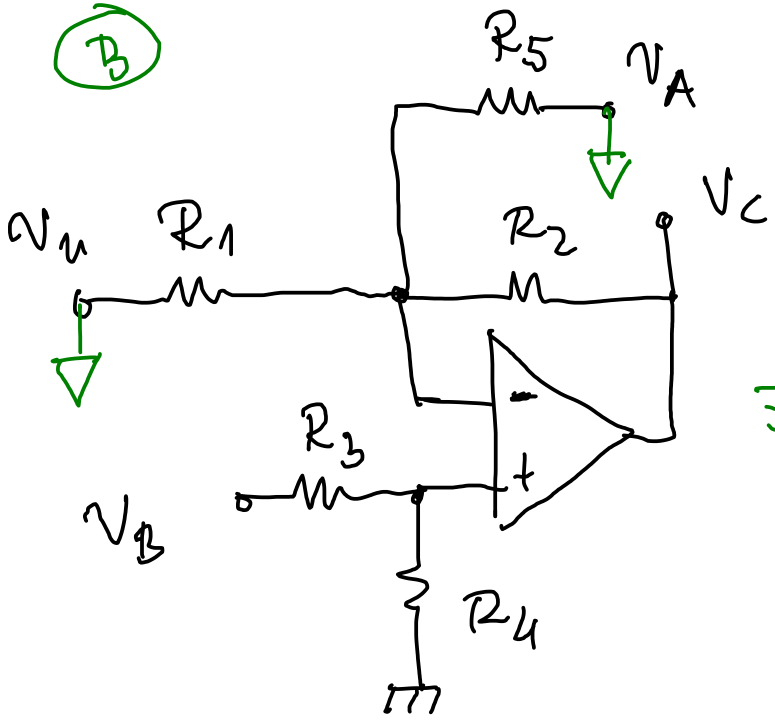
$$V_{C_A} = -\frac{R_2}{R_5} \cdot V_A$$

U



$$V_{C_u} = -\frac{R_2}{R_1} \cdot V_u$$

B



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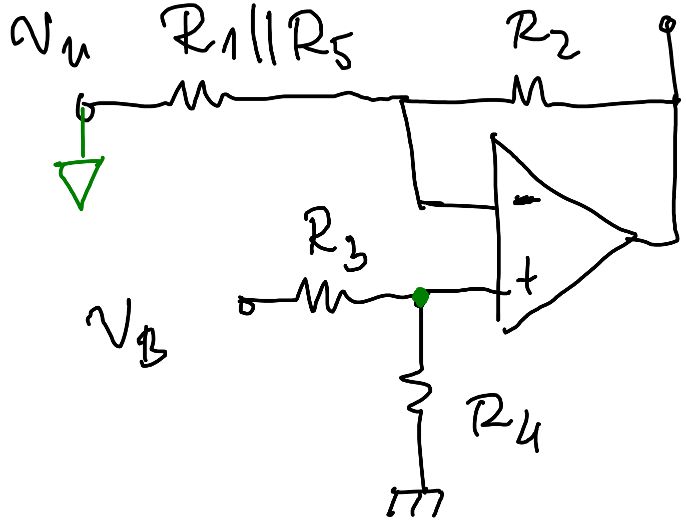

$$V_{C_B} = \left(1+\frac{R_2}{R_1\|R_5}\right)\frac{R_4}{R_4+R_3}\cdot V_B$$

$$V_C = V_{C_A}+V_{C_u}+V_{C_B} = -\frac{R_2}{R_5}V_A - \frac{R_2}{R_1}\cdot V_u + \overbrace{\left(1+\frac{R_2}{R_1\|R_5}\right)\cdot\frac{R_4}{R_4+R_3}}^{k}\cdot V_B$$

$$V_C = -\frac{R_2}{R_5}\,\frac{1}{s^2C_6R_6C_7R_7}\cdot V_C - \frac{R_2}{R_1}\cdot V_u - \frac{k}{sC_6R_6}V_C$$

$$H_C(s) = \frac{V_C}{V_{IN}} = \frac{-R_2/R_1 \quad \cdot \Big/ s^2 C_6 R_6 C_7 R_7 \cdot R_5/R_2}{1 + \frac{R_2}{R_5} \frac{1}{s^2 C_6 R_6 C_7 R_7} + \frac{k}{s C_6 R_6} \cdot \Big/ s^2 C_6 R_6 C_7 R_7 \cdot R_5/R_2}$$

$$H_C(s) = -\frac{R_2}{R_1} \cdot \frac{s^2 C_6 R_6 C_7 R_7 \cdot R_5/R_2}{1 + s\left[k \frac{R_5}{R_2} C_7 R_7\right] + s^2 C_6 R_6 C_7 R_7 \cdot R_5/R_2}$$ HPF

$$H_B(s) = \frac{V_B}{V_{IN}} = \frac{V_B}{V_C} \cdot \frac{V_C}{V_{IN}} = -\frac{1}{s C_6 R_6} \cdot H_C(s)$$ BPF

$$H_A(s) = \frac{V_A}{V_{IN}} = \frac{V_A}{V_C} \cdot \frac{V_C}{V_{IN}} = \frac{1}{s^2 C_6 R_6 C_7 R_7} \cdot H_C(s)$$ LPF